Commun. Math. Phys. 120, 295–333 (1988)

Communications in Mathematical Physics

4-Dimensional Black Holes from Kaluza-Klein Theories

Peter Breitenlohner[1], Dieter Maison[1], and Gary Gibbons[2]

[1] Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg-Institut für Physik, P.O. Box 401212, D-8000 München, Federal Republic of Germany

[2] Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, England, and Laboratoire de Physique Théorique de l'Ecole Normale Supérieure*, and Groupe d'Astrophysique Relativiste, Observatoire de Paris, Meudon, France

Abstract. In this paper we consider generalizations in 4 dimensions of the Einstein-Maxwell equations which typically arise from Kaluza-Klein theories. We specify conditions such that stationary solutions lead to non-linear σ-models for symmetric spaces. Using both this group theoretic structure and some properties of harmonic maps we are able to generalize many of the known existence and uniqueness theorems for black holes in Einstein-Maxwell theory to this more general setting.

1. Introduction

The advent of Einstein's non-linear theory of General Relativity seemed to open up the possibility of finding particle-like solutions of the classical equations of motion which are singularity free. However work by Serini [1], Einstein and Pauli [2], and Lichnerowicz [3] established that if a solution is asymptotically flat, topologically trivial and globally stationary (i.e. admits an everywhere time-like Killing vector field) then it must be flat space. In modern parlance one might say that Einstein's vacuum theory does not admit strictly stationary "soliton" solutions which are topologically trivial and without singularities.

In fact Lichernowicz's argument does extend to topologically non-trivial solutions of the form $\Sigma \times \mathbf{R}$, where Σ is a complete asymptotically flat spacelike surface with perhaps more than one asymptotic region, but it was necessary for him to assume that the Killing vector is everywhere time-like. The possibility of more than one asymptotic region did not appear explicitly in his work, but in view of the "wormhole" structure of the constant time surfaces of the Schwarzschild solution discovered by Einstein and Rosen [4] it is evidently necessary to allow for this case. At the time Einstein and Rosen appeared to believe that they had obtained a singularity free solution, but as the global structure of the Schwarzschild solution became better understood it became clear that there was indeed a

* Laboratoire Propre du Centre Nationale de la Recherche Scientifique, associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud