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Diamagnetic Currents

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Abstract. We study the diamagnetic surface currents of particles in thermal equilibrium submitted to a constant magnetic field. The current density of independent electrons with Boltzmann (respectively Fermi) statistics has a gaussian (respectively exponential) bound for its fall off into the bulk. For a system of interacting particles at low activity with Boltzmann statistics, the current density is localized near to the boundary and integrable when the two-body potential decays as $|\mathbf{x}|^{-\alpha}, \alpha > 4$, in three dimensions. In all cases, the integral of the current density is independent of the nature of the confining wall and correctly related to the bulk magnetisation. The results hold for hard and soft walls and all field strength. The analysis relies on the Feynman-Kac-Ito representation of the Gibbs state and on specific properties of the Brownian bridge process.

1. Introduction

It is well known that the diamagnetism of a system of charges at thermal equilibrium is a purely quantum mechanical effect. The magnetisation of a body subjected to an uniform magnetic field arises from induced currents localised at its surface. Classically, one argues that these currents should be exactly compensated by the cyclotronic motion of the particles in the bulk [1] and there is no diamagnetism in agreement with van Leuwen's theorem [2]. For quantum mechanical charges, this compensation is not perfect and there is a resulting current density near the boundaries which depends on the nature of the walls enclosing the system. However, the corresponding total magnetisation, a thermodynamical quantity, is independent of boundary effects.

In [3], the existence of the thermodynamic limit of the susceptibility of a system of non-interacting electrons (the Landau susceptibility) is rigorously established for all temperatures and densities. Moreover, the susceptibility is independent of the boundaries for a class of parallelepipeds with Dirichlet conditions, but its relation to

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