Commun. Math. Phys. 117, 191–202 (1988)

On Structural Stability for Semiflows

Juergen Quandt*

Division of Applied Mathematics, Brown University, Providence, RI, 02912, USA

Abstract. Some results on structural stability which are known to hold for flows of a compact manifold are extended to semiflows of a Banach space.

In this paper we consider the problem of structural stability for evolution equations which generate semiflows of Banach spaces. From a theoretical point of view it is advantageous in such a study to look at the semiflows directly without referring back to the generating equations. We will do this here by choosing a natural topology on the class of all semiflows of a given Banach space. We then ask the question under which conditions semiflows behave qualitatively alike if they are slightly perturbed in the above topology. Physically, this is motivated by the fact that, in experiments, semiflows can only be approximately determined.

Guided by the theory of flows, where the main assumption leading to structural stability is "hyperbolicity," we make the following observation. If we weaken both the concept of structural stability and hyperbolicity by restricting it to the global (bi-infinite) solutions, then the theory of flows carries over to the semiflow case. Physically, this may be justified by the fact that only those solutions seem to play a significant role in nature.

In Sects. 1 and 2 we extend results of Palis [6] and Pugh [7] on hyperbolic linear flows, and of Anosov [1] and Moser [5] on Anosov flows to the semiflow case, and Sect. 3 contains some ideas towards a general stability theorem for semiflows. A similar discussion for maps may be found in [8, 9].

0. Introduction

Let $(X, |\cdot|)$ be a Banach space. A C^r-semiflow of X, $r \geqq 0$, is a one-parameter family of C^r-maps $\{F(t): X \to X | t \geqq 0\}$ such that

1) $F(s+t) = F(s)F(t)$ for all $s, t \geqq 0$,
2) $F(0)$ is the identity,
3) $F(\cdot)(x)$: $[0, \infty[\to X$ is continuous for every $x \in X$.

---

* Current address: Department of Mathematics, University of Central Florida, Orlando, FL 32816, USA