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Extension of the Module of Invertible Transformations. Classification of Integrable Systems

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Abstract. We demonstrate that for the systems of equations, which are invariant under a point group or possess conservation laws of the zeroth or first order, a nontrivial extension of the module of invertible transformations is possible. That simplifies greatly a classification of the integrable systems of equations. Here we present an exhaustive list and a classification of the second order systems of the form $u_t = u_{xx} + f(u, v, u_x, v_x)$, $-v_t = v_{xx} + g(u, v, u_x, v_x)$, which possess the conservation laws of higher order. The reduction group approach allows us to define the Lax type representations for some new equations of our list.

Introduction

The systems of evolution equations, related by the invertible transformations, should be considered as equivalent ones. In many applications there occurs a situation when a system of equations possess a continuous point symmetry group, and it is sufficient to restrict ourselves to a reduced subset of dynamical variables, consisting of the group invariants. With the accuracy up to the unessential constants of integration, which can be removed by the transformations of the group, a reduced subset contains all the information about a general solution. Thus, we can also consider two systems of equations as equivalent ones, if their reduced subsets of dynamical variables are related by invertible substitutions. In contrast to the point transformations, such substitutions may violate local conservations laws. We shall study the most interesting substitutions, that preserve the locality property of the conservation laws (recall, the conservation law $\rho_t = D\sigma$ is called local, if ρ and σ are the functions of a finite number of the dynamical variables). The considered module extension of the invertible substitutions simplifies drastically the classification of the integrable equations. The use of these substitutions allowed us to make a list of integrable systems of equations [1, 2, 3] more comprehensible. Many of the well-known equations have proved to be equivalent by this extended