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## The Internal Symmetry Group of a Connection on a Principal Fiber Bundle with Applications to Gauge Field Theories

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Abstract. The internal symmetry group of a connection on a principal fiber bundle P is studied. It is shown that this group is a smooth proper Lie transformation group of P, which, if P is connected, is also free. Moreover, this group is shown to be isomorphic to the centralizer of the holonomy group of the connection. Several examples and applications of these results to gauge field theories are given.

## Contents

Introduction																	231
1. Notation and Prelim	inaries.																233
2. The Lie Group $I_{\omega}(P)$	)																240
3. Some Examples																	244
4. Connections with In	ternal S	ymme	etry	Gro	oup	as	Lar	ge	as	Рс	ossi	ble					250
5. Connections with In	ternal S	ymme	etry	Gro	oup	as	Sm	all	as	Po	ossi	ble					
and Irreducible Con	nections																252
6. $I_{\omega}(P)$ as a Lie Group	o of Trai	nsforr	nati	ons													256
7. The Lie Group of Tr	ransform	natior	is $I_i$	a, w)(	<i>P</i> ).												259
References				•													262

## Introduction

Let P(M, G) denote a principal fiber bundle with structure group G over a connected manifold M, let AUT(P) denote the group of automorphisms of P, and let Aut(P) denote the normal subgroup of automorphisms of P that cover the identity diffeomorphism of M. Let  $\omega$  be the connection 1-form of a connection on P. If  $F \in AUT(P)$ , let  $F^*\omega$  denote the pullback of  $\omega$  by F. Then

$$AUT_{\omega}(P) = \{F \in AUT(P) | F^*\omega = \omega\}$$

is the symmetry group of  $\omega$ , and

 $I_{\omega}(P) = \{F \in \operatorname{Aut}(P) | F^* \omega = \omega\} = \operatorname{Aut}(P) \cap \operatorname{AUT}_{\omega}(P)$