Commun. Math. Phys. 113, 209–230 (1987)

Communications in Mathematical Physics

Asymptotic Motion of a Classical Particle in a Random Potential in Two Dimensions: Landau Model***

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Abstract. We consider the motion of a classical particle in a random isotropic potential arising from uniformly distributed scatterers in two dimensions. We prove that in the weak coupling limit the velocity process of the particle converges in distribution to Brownian motion on a surface of constant speed, i.e. on the circle. The resulting equation for the probability density of the particle is related to the Landau equation in plasmas.

Introduction

We consider the classical motion of a point particle in a random potential $U(\mathbf{x})$ in the van Hove limit: Let $U_\omega(\mathbf{x})$ be a realization of the random potential and let $\mathbf{x}(t)$, $\mathbf{v}(t)$ denote position and velocity of the particle given by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{v}(t), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ \dot{\mathbf{v}}(t) &= -\varepsilon \nabla U_\omega(\mathbf{x}(t)) = \varepsilon \mathbf{F}_\omega(\mathbf{x}(t)), \quad \mathbf{v}(0) = \mathbf{v}_0. \end{aligned}$$

Clearly $(\mathbf{x}(t), \mathbf{v}(t))_{t \geqq 0}$ is a stochastic process (on the probability space of the random potential). We study the distribution of

$$(\mathbf{x}^\varepsilon(t), \mathbf{v}^\varepsilon(t))_{t \geqq 0}, \dot{\mathbf{x}}^\varepsilon(t) = \mathbf{v}^\varepsilon(t), \mathbf{v}^\varepsilon(t) = \mathbf{v}(t/\varepsilon^2)$$

as $\varepsilon \to 0$.

Kesten and Papanicolaou showed that under some mixing assumption on the (non-conservative) force $\mathbf{F}$ the processes $(\mathbf{x}^\varepsilon(t), \mathbf{v}^\varepsilon(t))_{t \geqq 0}$ converge in distribution to a diffusion process $(\mathbf{x}^{0'}(t), \mathbf{v}^0(t))_{t \geqq 0}$ as $\varepsilon \to 0$ when the space dimension is larger than 2 [1]. (See [2] for a quantum mechanical version.) The two dimensional case was left open.

The dimensionality comes into play since in three or higher dimensions trajectories $(\mathbf{x}^0(t))$ of the limit process do not intersect themselves, whereas in the

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***Work supported in part by National Science Foundation Grant No. DMS-85-12505 and AFOSR No. C010