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Second Order Large Deviation Estimates for Ferromagnetic Systems in the Phase Coexistence Region*

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Abstract. We consider the d-dimensional Ising model with ferromagnetic nearest neighbor interaction at inverse temperature β. Let $M_\Lambda = |\Lambda|^{-1} \sum_{i\in\Lambda} \sigma_i$ be the magnetization inside a d-dimensional hyper cube Λ, μ_+ be the + Gibbs state and $m^*(\beta)$ be the spontaneous magnetization. For β such that $m^*(\beta) > 0$ we find a sufficient condition (easily verified to hold for large β) for $\mu_+(\{M_\Lambda \in [a, b]\})$ to decay exponentially with $|\Lambda|^{(d-1)/d}$ when $-m^* < b < m^*$, $-1 \leqq a < b$. For $d = 2$ this sufficient condition is the exponential decay of a connectivity function. We also prove a partial converse to this result, obtain a sharper result for the magnetization on $d-1$ dimensional cross sections of the model and prove a similar result for $d = 2$, $-m^* < a < b < m^*$, and β large, when free boundary conditions are chosen outside Λ.

1. Introduction

We consider the Ising model with nearest neighbor interaction on a d-dimensional lattice Z^d. The spin at each point $x \in Z^d$ takes the value $\sigma_x = \pm 1$, and the formal energy of a spin configuration σ is

$$E(\sigma) = -(1/2) \sum_{x,y} J_{x,y} \sigma_x \sigma_y,$$

where $J_{x,y} = 1$ if x and y are nearest neighbors and $J_{x,y} = 0$ otherwise. A Gibbs measure at inverse temperature β is any measure μ on $\{-1, 1\}^{Z^d}$ such that for any choice of $a_y = \pm 1$, $y \in Z^d$, and any $x \in Z^d$

$$\begin{aligned}&\mu(\{\sigma : \sigma_x = a_x\} \mid \{\sigma : \sigma_y = a_y \text{ for } y \neq x\}) \\ &\quad = \exp((\beta/2) \sum_y J_{x,y} a_x a_y) / (\exp((\beta/2) \sum_y J_{x,y} a_y) + \exp(-(\beta/2) \sum_y J_{x,y} a_y)),\end{aligned}$$

μ almost surely.

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