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Dirac and Majorana Spinors on Non-Orientable Riemann Surfaces*

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Abstract. It is well known that (Weyl) spinors cannot be consistently defined on nonorientable manifolds. We prove that Dirac spinors can be defined on nonorientable Riemann surfaces. It is also shown that Majorana spinors cannot be defined consistently on closed nonorientable Riemann surfaces with odd Euler number, but can be consistently defined in all other cases.

The recent surge of interest in string theories was triggered by the demonstration of the one loop cancellation of anomalies in the type I $SO(32)$ superstring [1]. This calculation was carried out using the canonical quantization procedure. Green and Schwarz had already realized that similar infinity cancellations must be operative at tree level, but they could not prove the vanishing of the dilaton tadpole at this level since it was not clear how the canonical quantization procedure could be used to obtain this result. The alternative path integral quantization method of Polyakov can be used for this purpose [2]. In fact, it was recently used to show the vanishing of the dilaton tadpole for the $SO(8192)$ type I bosonic string theory [3].

The path integral quantization procedure for field theories is notorious for its suitability in discussing global obstructions to the consistent formulation of the theory. For example, Witten has shown that a class of gauge theories with spinors is inconsistent [4], but the corresponding anomaly is much more difficult to understand if canonical quantization methods are used [5]. Polyakov's formulation of the super-string can then be expected to shed new light on possible global inconsistencies of superstring theories.

The partition function in Polyakov's theory is a sum over two-dimensional field theories defined on different Riemann surfaces. For the type I superstring, it includes both open and closed, oriented and nonoriented surfaces. The formulation is in Euclidean space and the action is just the integral of the supergravity

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