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Compact Matrix Pseudogroups

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Abstract. The compact matrix pseudogroup is a non-commutative compact space endowed with a group structure. The precise definition is given and a number of examples is presented. Among them we have compact group of matrices, duals of discrete groups and twisted (deformed) $SU(N)$ groups. The representation theory is developed. It turns out that the tensor product of representations depends essentially on their order. The existence and the uniqueness of the Haar measure is proved and the orthonormality relations for matrix elements of irreducible representations are derived. The form of these relations differs from that in the group case. This is due to the fact that the Haar measure on pseudogroups is not central in general. The corresponding modular properties are discussed. The Haar measures on the twisted $SU(2)$ group and on the finite matrix pseudogroup are found.

0. Introduction

Let G be a Lie group. A family $(G_\tau)_{\tau\in[0,\varepsilon[}$ of Lie groups is said to be a deformation of G if $G_0 = G$ and G_τ depends continuously on τ. The latter should be understood in a natural sense. For example one may require that all G_τ are of the same dimensions and that it is possible to choose bases in g_τ (g_τ is the Lie algebra of G_τ) such that the corresponding structure constants depend continuously on τ.

Assume that the group G is involved in a theory (e.g. it is a symmetry group) describing a physical reality. As we well know any physical theory describes well only a limited class of phenomena, for the phenomena beyond this class the theoretical predictions disagree with the experimental results. In order to obtain the adequate description of a larger class of phenomena one must modify the theory. In certain cases such a modification although revolutionary from the conceptual point of view consists in replacing G by one of the group G_τ. Then the value of τ is one of the fundamental constants (small parameter) of the new, more general theory. Within this new theory the group G retains its validity only in the approximate sense (e.g. it describes a broken symmetry). The old theory can be recovered in the limit $\tau\to 0$.