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The Diffusion of Self-Avoiding Random Walk in High Dimensions

Gordon Slade*

Department of Mathematics, University of Virginia, Charlottesville, VA 22903, USA

Abstract. We use the Brydges-Spencer lace expansion to prove that the mean square displacement of a T step strictly self-avoiding random walk in the d dimensional square lattice is asymptotically of the form DT as T approaches infinity, if d is sufficiently large. The diffusion constant D is greater than one.

1. Introduction

A T step self-avoiding walk on the d dimensional square lattice $\mathbb{Z}^d$ is a set of $T+1$ points $\omega(0)=0, \omega(1), \omega(2), \ldots, \omega(T)$ in $\mathbb{Z}^d$ with $|\omega(i+1)-\omega(i)|=1$ and $\omega(i) \neq \omega(j)$ for $i \neq j$. A probability measure is defined on the set of all T step self-avoiding walks by assigning an equal probability to each such walk. Numerical and other evidence suggests that the mean square displacement with respect to this measure, i.e., the expected value $\langle \omega(T)^2 \rangle$ of $\omega(T) \cdot \omega(T)$, is asymptotically of the form DT^α as $T \to \infty$, where $\alpha=1.5$ for $d=2$, $\alpha=1.18$ for $d=3$, $\alpha=1$ with logarithmic corrections for $d=4$, and $\alpha=1$ for $d \geqq 5$ [4]. For $d=1$ there are only two self-avoiding walks, $\langle \omega(T)^2 \rangle = T^2$, and $\alpha=2$. Removing the self-avoidance constraint $\omega(i) \neq \omega(j)$, $i \neq j$ gives the simple random walk, for which $\langle \omega(T)^2 \rangle = T$ in all dimensions.

In spite of the apparent simplicity of the self-avoiding walk model, apart from the result obtained below there is no rigorous proof that α is as stated above. In this paper we prove that $\alpha=1$ and $D>1$ for $d \geqq d_0$, for some $d_0 \geqq 5$. No effort has been made to obtain the best possible value of d_0. It is not surprising that $D>1$ here, since it is to be expected that a self-avoiding walk will on the average end up farther away from the origin than a simple walk.

Other results for the critical exponents of self-avoiding random walk can be found in [7, 8]. In [8] the connection between self-avoiding walk and quantum field theory is also explained. Lawler [6] considered a related model, the loop-erased self-avoiding random walk, and proved that for $d \geqq 4$ scaled loop-erased

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* Present address: Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S 4K1