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The Lax Representation for an Integrable Class of Relativistic Dynamical Systems

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Abstract. We exhibit the Lax pair for the class of relativistic dynamical systems recently introduced by Ruijsenaars and Schneider, whose equations of motion read $\ddot{q}_j = \sum_{k=1, k \neq j}^{n} \dot{q}_j \cdot \dot{q}_k \cdot v(q_j - q_k)$, $j = 1, 2, \ldots, N$, with $v(x) = \mathscr{P}'(x)/[B - \mathscr{P}(x)]$, where B is an arbitrary constant and $\mathscr{P}(x)$ the Weierstrass elliptic function.

1. Introduction

Recently Ruijsenaars and Schneider [1] have introduced a class of integrable dynamical systems, whose interest is underlined by the possibility to interpret these models as describing the one-dimensional motion of n interacting relativistic particles and by their relation to the multisoliton solutions of certain integrable PDEs such as the sine-Gordon equation. The main purpose of this paper is to exhibit the representation of the equations of motion of these systems,

$$\ddot{q}_j = \sum_{\substack{k=1 \\ k \neq j}}^{n} \dot{q}_j \dot{q}_k v(q_j - q_k), \qquad q_j = q_j(t), \qquad j = 1, 2, \ldots, n, \tag{1.1}$$

in the Lax form [2],

$$\dot{\mathbf{L}} = [\mathbf{L}, \mathbf{M}], \tag{1.2}$$

where $\mathbf{L}$ and $\mathbf{M}$ are the $n \times n$ matrices,

$$L_{jk} = \delta_{jk} \dot{q}_j + (1 - \delta_{jk}) (\dot{q}_j \dot{q}_k)^{1/2} \alpha(q_j - q_k), \tag{1.3}$$

$$M_{jk} = \delta_{jk} \sum_{\substack{m=1 \\ m \neq j}}^{n} \dot{q}_m \beta(q_j - q_m) + (1 - \delta_{jk}) (\dot{q}_j \dot{q}_k)^{1/2} \gamma(q_j - q_k). \tag{1.4}$$

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