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Diffusion Processes with Singular Drift Fields

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Abstract. A class of stochastic differential equations with highly singular drift fields is considered. Using a purely probabilistic approach, we can show the unattainability of the nodal set. Moreover, a global existence and uniqueness theorem for diffusion processes with singular drift fields is established. The finite action condition of Carlen and Zheng can be modified. We relate our results to the diffusions which describe the time evolution of quantum systems in stochastic mechanics.

I. Introduction and Summary

The main bulk of the mathematical literature on stochastic differential equations (SDEs) tackles the question of existence and uniqueness of solutions under assumptions very reminiscent of those for deterministic (ordinary or partial) differential equations (see e.g. [1–3]). Usually the coefficients of the SDEs (the so-called infinitesimal characteristics) are required to satisfy some regularity condition (such as a Lipschitz condition) to ensure local existence and uniqueness of a continuous solution, and a growth condition is imposed to avoid explosions, i.e. to prevent the process from running off to infinity within finite time.

Both from a mathematical point of view and a look towards applications in other disciplines, e.g. physics or biology, it is desirable to relax the standard conditions.

There has been some previous work in this field. The stationary case was first considered by Albeverio and Høegh-Krohn [4], and then by Carmona [5], Nagasawa [6], and Albeverio et al. [7]. The analysis in [5, 7] is in terms of Dirichlet forms. A nice feature of this work is that the probability densities involved are admitted to be discontinuous. The point of view of Dirichlet forms is also taken in papers by Albeverio et al. [8], and Fukushima [9]. Reference [8] also discusses several physical aspects of the unattainability of the nodal set (i.e. the set

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