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# Conformally Invariant Differential Operators on Minkowski Space and Their Curved Analogues

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**Abstract.** This article describes the construction of a natural family of conformally invariant differential operators on a four-dimensional (pseudo-)Riemannian manifold. Included in this family are the usual massless field equations for arbitrary helicity but there are many more besides. The article begins by classifying the invariant operators on flat space. This is a fairly straightforward task in representation theory best solved through the theory of Verma modules. The method generates conformally invariant operators in the curved case by means of Penrose's local twistor transport.

## Introduction

Special relativistic field equations are those which are invariant under the Poincaré group. The Poincaré invariance of Maxwell's equations may be regarded as the starting point for the special theory of relativity. Maxwell's equations and other *massless* field equations can, however, be made to exhibit [2, 7] an even larger invariance group, namely the *conformal group* of Minkowski space. Our first objective in this article is to describe all conformally invariant differential operators on Minkowski space.

Our second objective is to exhibit precise analogues of these differential operators on space-times which are not flat. For this, the sense of the analogy is the important point since there is no longer a transitive group of conformal transformations acting on a curved space-time; invariance under a conformal group becomes meaningless. However, for differential operators depending on a choice of metric there is the concept of invariance under *conformal rescaling* of the metric, i.e. replacing the metric $g_{ab}$ with a metric $\hat{g}_{ab} = \Omega^2 g_{ab}$ for $\Omega$ a nowhere-vanishing function. If $D$ is a differential operator corresponding to $g_{ab}$ then denote by $\hat{D}$ the operator corresponding to the rescaled metric $\hat{g}_{ab}$. One allows tensors, etc., to be rescaled by powers of $\Omega$, so that if we decide to put $\hat{\phi} = \Omega^r \phi$, then we say that $\phi$ is

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