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Special Energies and Special Frequencies

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Abstract. "Special frequencies" have been asserted to be zeros of the density of frequencies corresponding to a random chain of coupled oscillators. Our investigation includes both this model and the random one-dimensional Schrödinger operator describing an alloy or its discrete analogue. Using the phase method we exactly determine a bilateral Lifšic asymptotic of the integrated density of states $k(E)$ at special energies E_s, which is not only of the classical type $\exp(-c/|E-E_s|^{1/2})$ but also $\exp(-c'/|E-E_s|)$ is a typical behaviour. In addition, other asymptotics occur, e.g. $|E-E_s|^{c''}$, which show that $k(E)$ need not be C^∞.

1. Introduction

In this paper, we consider the random Schrödinger operator (Hamiltonian)

$$H^\omega = -\frac{d^2}{dx^2} + V^\omega(x) \quad \text{on} \quad L^2(R) \tag{1}$$

with

$$V^\omega(x) = \sum_{n\in Z} V_{\omega(n)}(x-n), \tag{2}$$

where the indices $\omega(n)$ are random variables on the realization space Ω with values in the set $\{1,2,\ldots,r\}$. We deal with the case in which the random process $\omega(n)$ is independent, identically distributed and the functions V_i are form-bounded with respect to $-d^2/dx^2$ (e.g., they can be bounded or δ-functions, cf. [1]) and satisfy $\operatorname{supp} V_i \subseteqq [0,1)$ for all $i\in\{1,2,\ldots,r\}$ (the random process can be chosen more generally, cf. [2]). The operator H^ω defined in this way describes a one-dimensional r-ary random alloy in the one-body approximation.

Our interest is directed to the integrated density of states, i.e. to the limit

$$k(E) = \lim_{L\to\infty} \frac{1}{L} N_E(H_L^\omega), \tag{3}$$