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Highest Weight Representations of the Neveu-Schwarz and Ramond Algebras

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Abstract. We construct a family of representations $\mathscr{H}^{\xi,w}$ of the Neveu-Schwarz and Ramond algebras, which generalize the Fock representations of the Virasoro algebra. We show that the representations $\mathscr{H}^{\xi,w}$ are intertwined by a vertex operator.

The above results are used to give the proof of the conjectured formulas for the determinant of the contravariant form on the highest weight representations of the Neveu-Schwarz and Ramond algebras. Further results on the representation theory of the latter are derived from the determinant formulas.

1. Introduction

In Superstring theory physicists consider two supersymmetric extensions of the Lie algebra of vector fields on the circle ($\mathrm{Vect}(S^1)$) called the Neveu-Schwarz [19] and Ramond [20] algebras. The Neveu-Schwarz algebra has basis $\{L'_0, L_i, G_j\}$ $(i \in \mathbb{Z}, j \in \frac{1}{2} + \mathbb{Z})$, where L'_0 is central, and the bracket of two noncentral generators is given by the relations

$$[L_i, L_j] = (j-i)L_{i+j} + \delta_{i,-j}\left(\frac{i^3-i}{8}\right)L'_0\,,$$

$$[G_i, G_j] = -2L_{i+j} + \delta_{i,-j}\tfrac{1}{2}(i^2-\tfrac{1}{4})L'_0\,,$$

$$[L_i, G_j] = (j-\tfrac{1}{2}i)G_{i+j}\,.$$

The Ramond algebra has the same relations, but the G_j are indexed by $\mathbb{Z}$. These algebras are "$\mathbb{Z}_2$-graded Lie algebras," i.e., $\mathbb{Z}_2$-graded vector spaces with a grading

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