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The Limiting Absorption and Amplitude Principles for the Diffraction Problem with Two Unbounded Media

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Abstract. We consider the classical diffraction problem for the wave propagation in the case where the propagation speed is piecewise constant, and the surface separating two media is unbounded. The validity of the limiting absorption and amplitude principles is proved.

1. Introduction

We deal with the asymptotic behaviour (as $t \to +\infty$) of the following Cauchy problem

$$\mu(x) w_{tt} - \Delta w = e^{-i\omega t} f(x), \tag{1.1}$$

$$w|_{t=0} = w_t|_{t=0} = 0, \tag{1.2}$$

in the space $\mathbb{R}^n = \{x\}, x = (x_1, \ldots, x_n)$ (we denote the radius vector of the point x by the same letter), where $n \geqq 3, \omega = \text{const} > 0, f$ belongs to some L^2-weighted space,

$$\mu(x) = \frac{1}{a^2(x)}.$$

$a(x)$ is a wave propagation speed.

We assume that $\mu(x)$ has only two values:

$$\mu(x) = \mu_j \quad \text{if } x \in \Omega_j, \quad j = 1, 2 \tag{1.3}$$

where $\mu_j = \text{const} > 0$,

$$\Omega_1 = \{x : x \in \mathbb{R}^n, \quad x_n > \varphi(\tilde{x})\},$$

$$\Omega_2 = \{x : x \in \mathbb{R}^n, \quad x_n < \varphi(\tilde{x})\},$$

where $\tilde{x} = (x_1, \ldots, x_{n-1}) \in \mathbb{R}^{n-1}$, $\varphi(\tilde{x})$ is some given function, defined on $\mathbb{R}^{n-1}$ and satisfying the condition

$$\varphi \in C^1(\mathbb{R}^{n-1} \backslash \{0\}). \tag{1.4}$$