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# Spinors and Diffeomorphisms

L. Dąbrowski* and R. Percacci
International School for Advanced Studies, SISSA, I-34014 Trieste, Italy

**Abstract.** We discuss the action of diffeomorphisms on spinors on an oriented manifold $M$. To do this, we first describe the action of the diffeomorphism group $D(M)$ on the set $\Pi = H^1(M, Z_2)$ of inequivalent spin structures and show that it is affine. We argue that in the presence of spinors the gauge group of gravity is a certain double cover of $D(M)$ which depends on the spin structure. We explicitly compute the action of $D(M)$ on $\Pi$ when $M$ is a closed Riemann surface; $\Pi$ is seen to consist of exactly two orbits, corresponding to even and odd spin structures.

## 1. Introduction

It is often said that spinor fields transform as scalars under diffeomorphisms and as "spinors" under local rotations of the orthonormal frames. While this statement is true at the level of local components, it does not specify the transformation behaviour of the spinors regarded as intrinsic geometric objects. Therefore, in handling global geometric properties of the spinors, anomalies and similar problems, it is convenient to have a coordinate-free description of the action of the diffeomorphism group. This is more complicated than the action of the diffeomorphism group on tensors, for the following reason. The tensorfields on a manifold form an infinite dimensional linear space and diffeomorphisms transform this linear space into itself. On the other hand, the definition of spinorfields on a manifold $M$ requires a previous specification of a metric tensor; for each metric tensor there is a distinct space of spinorfields. There is no natural way of identifying these spaces. Therefore, the configuration space of coupled spinors and metric tensors is not the cartesian product of the separate configuration spaces but rather an infinite dimensional vectorbundle $\hat{\mathscr{W}}$ over the configuration space of the metric tensors. The fiber of this bundle over a particular metric tensor $g$ is precisely the space of spinors for $g$. Since diffeomorphisms transform the metric tensor by

* On leave of absence from I.F.T., University of Wrocław, Poland