Commun. Math. Phys. 106, 679–689 (1986)

Communications in Mathematical Physics

Ornstein-Zernike Theory for the Planar Random Surface Model

Thordur Jonsson

The Science Institute, University of Iceland, Dunhaga 3, 107 Reykjavik, Iceland

Abstract. We prove that the two-loop function in the planar random surface (PRS) model has Ornstein-Zernike decay for all noncritical values of the temperature. A notion of breathing is introduced and it is shown that surfaces do not breathe at noncritical temperatures. With the aid of a simple assumption, supported by mean field theory and numerical calculations, we prove that the scaling limit of the PRS-model exists and equals that of a free field.

1. Introduction and Results

Lattice gauge theories, string theory and various problems in statistical mechanics have recently stimulated interest in random surface theories, see [8–10] for review of some of the recent work. By now a considerable amount of information about a few models of lattice surfaces has been obtained. One of these is the model of SOS-tubes, studied by Abraham, Chayes and Chayes (ACC) [1–3]. Another one is the PRS-model studied by Eguchi, Kawai and Okamoto [12, 13] and in more detail by Durhuus, Fröhlich and Jónsson (DFJ) [4–7].

ACC have developed powerful methods, based on earlier work on the theory of classical fluids [14, 15], that enable them to analyze completely the noncritical behaviour of SOS-tubes. DFJ have calculated all the critical exponents in the PRS-model [6] with the aid of a simple plausible hypothesis which is consistent with numerical simulations [11, 12] and mean field theory [6, 12]. For other recent work on random surface models, see [10] and references therein.

The purpose of this paper is to show that the methods of ACC apply to the PRS-model. In particular, one can calculate power corrections to the exponential decay of the loop-loop correlation function and show that it has Ornstein-Zernike decay. Bricmont and Fröhlich have established Ornstein-Zernike decay for correlations in the self-avoiding surface model by different methods [16]. Furthermore, it is shown that the scaling limit of the two-loop function exists if the aforementioned hypothesis of [6] is true. The scaling limit equals that of simple random walk. This behaviour was strongly suggested by the results of [6]. Finally,