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How Often do Simple Dynamical Processes Have Infinitely Many Coexisting Sinks?

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Abstract. This work concerns the nature of chaotic dynamical processes. Sheldon Newhouse wrote on dynamical processes (depending on a parameter μ) $x_{n+1} = T(x_n; \mu)$, where x is in the plane, such as might arise when studying Poincaré return maps for autonomous differential equations in $\mathbb{R}^3$. He proved that if the system is chaotic there will very often be existing parameter values for which there are infinitely many periodic attractors coexisting in a bounded region of the plane, and that such parameter values μ would be dense in some interval. The fact that infinitely many coexisting sinks can occur brings into question the very nature of the foundations of chaotic dynamical processes. We prove, for an apparently typical situation, that Newhouse's construction yields only a set of parameter values μ of measure zero.

1. Posing the Problem

Sheldon Newhouse made some amazing discoveries about dissipative dynamical processes. His discoveries have raised questions about the nature of systems that behave chaotically. The objective of this paper is to make computations that interpret his results, and to help re-establish the foundation of chaotic dynamical processes.

We are accustomed to thinking of dynamical systems with one or several basins of attraction. For our purposes a periodic point p of period k for a map T is called *attracting* if all the eigenvalues of $D(T^k(p))$ are inside the unit circle in the complex plane, where the operator D is the matrix of partial derivatives and T^k is the k^{th} iterate of T. Newhouse has shown it is possible for T to have infinitely many

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