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A Rigorous Replica Trick Approach to Anderson Localization in One Dimension

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Abstract. Let $H = -\Delta + V$ on $l^2(\mathbb{Z})$, where $V(x)$, $x \in \mathbb{Z}$, are i.i.d.r.v.'s, and let $G_L(x, y; E + i\eta) = \langle x|(H_L - (E + i\eta))^{-1}|y\rangle$, where H_L denotes the operator H restricted to $\{-L, -L+1, \ldots, L\}$ with Dirichlet boundary conditions. We use a supersymmetric replica trick to prove that

$$\mathbf{E}(|G_L(0, x; E + i\eta)|^2) \leqq K\eta^{-2} \exp\{-m|\log\eta|^{-\sigma}|x|\}$$

for some $m > 0$, $\sigma > 0$, $K < \infty$, uniformly in L and E. This estimate, together with the usual necessary estimate on the density of states, implies zero conductivity and gives exponential localization by the Fröhlich, Martinelli, Scoppola, and Spencer method.

1. Introduction

The replica trick has been used in the physics literature to study the Anderson model by field theoretic methods [1]. It expresses the Green's function (or squared modulus of the Green's function) of the random Hamiltonian as a two-point function (or four-point function) of a field theory with n independent replicas, which is then averaged over the randomness of the potential (introducing a coupling between the replicas). The quantities of interest are calculated, and then the limit is taken as the number of replicas $n \to 0$.

This $n \to 0$ limit is very mysterious. To circumvent it, Parisi and Sourlas [2] and McKane [3] introduced the supersymmetric replica trick, in which both Bose fields ("commuting variables") and spinless Fermi fields ("anti-commuting variables") are used in equal number of replicas, giving an effective total number of replicas $n = 0$ without any need of taking a limit. This has been applied to the Anderson model [4].

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