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Global Existence of Generalized Solutions of the Spherically Symmetric Einstein-Scalar Equations in the Large

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Abstract. In this paper we study the global initial value problem for the spherically symmetric Einstein-scalar field equations in the large. We introduce the concept of a generalized solution of our problem, and, taking as initial hypersurface a future light cone with vertex at the center of symmetry, we prove, without any restriction on the size of the initial data, the global, in retarded time, existence of generalized solutions.

Section 0. Introduction

In [1] we began the study of the global initial value problem for Einstein's equations in the spherically symmetric case with a massless scalar field as the material model. In terms of a radial coordinate r and a retarded time coordinate u, whose level surfaces are future light cones with vertices at the center of symmetry, the spacetime metric has the form

$$ds^2 = -e^{2\nu}du^2 - 2e^{\nu+\lambda}dudr + r^2 d\Sigma^2,$$

where $d\Sigma^2$ is the metric of the standard 2-sphere. We reduced Einstein's equations to a single nonlinear evolution equation for the function $h = \partial(r\phi)/\partial r$, where ϕ is the matter field. If f is a function of u and r we denote by $\bar{f}$ the mean value function of f:

$$\bar{f}(u,r) := (1/r)\int_0^r f(u,r')dr'.$$

Then, letting

$$g := \exp\left[-4\pi\int_r^\infty (h-\bar{h})^2\frac{dr}{r}\right]$$

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