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Generalized Solutions of the Radiative Transfer Equations in a Singular Case

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Abstract. This paper is devoted to the study of the radiative transfer equations:

$$\text{(TR)}\qquad \begin{cases} \dfrac{\partial \mathscr{E}}{\partial t} + \displaystyle\int_{\nu,\Omega} \sigma_\nu(\mathscr{E})\,(B_\nu(\mathscr{E})-I)\,\frac{d\Omega}{|S^N|}\,d\nu = 0\,, \\ \dfrac{\partial I}{\partial t} + \Omega\cdot\nabla_x I + \sigma_\nu(\mathscr{E})\,(I-B_\nu(\mathscr{E})) = \mathscr{D}I\,. \end{cases}$$

First, we prove a global existence theorem, which allows a blow-up of the opacity $\sigma_\nu(\mathscr{E})$ when $\mathscr{E}\to 0$. Thus, it extends Mercier's previous result [13]. This proof relies mainly on a nonlinear version of Hille-Yosida theorem: see Crandall-Ligett [9].

Then, we prove the uniqueness of the semigroup solving (TR), and some regularity results (in the class of functions with bounded variation).

Finally, we prove the convergence of some splitting algorithms associated to (TR).

Introduction

We are interested in a system of two nonlinear PDEs which can be actually regarded as a perturbation of the well-known transport equation. These equations are classical in astrophysics and represent the evolution of a stellar atmosphere in the absence of hydrodynamical motion and heat conduction. The photons in the medium will be ruled by a classical transport equation involving terms describing emission, true absorption and Thomson scattering. On the other hand we shall assume local thermodynamical equilibrium for the matter. It means that a local temperature T [and energy $\mathscr{E}(T)$] can be defined at each point of the medium. Moreover, the emission coefficient at each point is proportional to the true

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