Commun. Math. Phys. 105, 675–690 (1986)

Communications in
Mathematical
Physics

Deconfining Phase Transition in the U(1) Model with Wilson's Action

Chris King[†]
Department of Physics, Princeton University, Princeton, NJ 08544, USA

Abstract. In dimension $d \geqq 4$, the lattice U(1) gauge theory defined with the Wilson action is shown to have a deconfining phase transition at weak coupling. The proof uses a higher dimensional analogue of the Higgs mechanism and a correlation inequality to remove the massless modes of the theory. The remaining modes are controlled by a simple cluster expansion.

1. Introduction

This paper presents a new proof of the deconfining phase transition in the lattice U(1) gauge theory in four or more dimensions. Although this result is not new, our method of proof is quite novel. Furthermore, the proof uses the Wilson form of the action rather than the Villain form which appeared in earlier proofs [7, 4].

In the lattice formulation of gauge theories promoted by Wilson [10], an element of the gauge group is introduced on each bond of a d-dimensional lattice. The ordered product of group elements on the bonds around each plaquette p is written as $\mathrm{U}(p)$, and the Wilson form of the action is defined by

$$S_W = -\beta \sum_p \mathrm{Re}\,\mathrm{Tr}\,\mathrm{U}(p)\,. \tag{1.1}$$

This lattice theory is now a well-defined model in statistical mechanics, and may be analysed in a non-perturbative way. In this paper we shall examine the abelian model, for which the gauge group is U(1). So we write the group element on bond b as $\exp[iA(b)]$, where $A(b) \in [-\pi, \pi)$. Then $\mathrm{U}(p)$ is $\exp[iF(p)]$, where $F(p)$ is the sum over the oriented bonds in plaquette p of the field $A(b)$. The action (1.1) becomes

$$S_W = -\beta \sum_p \cos F(p)\,. \tag{1.2}$$

We are interested in the phase structure of the theory defined by (1.2). To analyse this, we will consider the expectation of a Wilson loop observable

[†] Research supported in part by U.S. National Science Foundation grant PHY 8117463