Commun. Math. Phys. 105, 511–518 (1986)

Communications in Mathematical Physics

Super-Kac-Moody Algebras and Supersymmetric $2d$-Free Fermions*

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Abstract. Explicit representations of super-Kac-Moody algebra are constructed in terms of $2d$-free fermions which form a non-linear representation of supersymmetry with the fermions grouped with the generators of the algebra into superfields. It is shown how the most general construction of this type corresponds to homogeneous spaces G/H and how supersymmetry alone is responsible for that structure.

It is well known that representations of Kac-Moody algebra [1] can be constructed using two-dimensional free fermions [2]. This construction was crucial in the proof by Witten [3] of the equivalence between non-linear sigma models with a Wess-Zumino term [4] and free fermion systems. This equivalence was later developed in a beautiful paper by Knizhnik and Zamolodchikov [5] using the techniques of conformal field theory [6]. It was then noticed that the supersymmetric extension of the sigma model [7] also had a rich algebraic structure and that it gave a representation of a supersymmetric extension of the Kac-Moody algebra [8]. In the case of SO(N) for example the content of the model in terms of free fermions is the following: there are two types of decoupled fields, one transforming under the adjoint representation of the group while the other (corresponding to the fermionization of the bosonic field of the original model) is in the fundamental representation. These two fields form a nonlinear representation of supersymmetry [8]. A similar property was also observed in Goddard and Olive in [9]. The purpose of this note is to show that a large class of representations of super-Kac-Moody algebra can be constructed in terms of free fermions which realize a non-linear representation of the two dimensional superconformal (Neveu et al. [10]) algebra (for another point of view on this latter construction and related considerations about superstrings see [11]). As we will

* This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contracts DE-AC03-76SF00098