Commun. Math. Phys. 105, 443–453 (1986)

Communications in Mathematical Physics

Methods of Topological Obstruction Theory in Condensed Matter Physics

I. A. Ovid'ko[1] and A. E. Romanov[2]

[1] Physico-Mechanical Department, M. I. Kalinin Polytechnical Institute, SU-195251 Leningrad, USSR
[2] Theoretical Department, A. F. Ioffe Physico-Technical Institute, SU-194021 Leningrad, USSR

Abstract. The notion of relative topological textures – nonuniform states of the general type in condensed (ordered) media – is introduced. For the classification of such states, an effective method is proposed which is based on the topological obstruction theory. The examples of relative topological textures are examined within the framework of the approach studied here.

I. Introduction

Topological methods are now recognized as being of great importance in condensed matter physics (for a review, see [1–4]) as well as in gauge theories of elementary particles (e.g. [5–8]). The former application of the homotopic topology allows one to classify defects and topological textures in condensed media in such a way that a large (but finite) energy barrier exists for the transformation of a defect (or a topological texture) from one class into another. Such media as ferromagnets, crystals, liquid crystals, superconductors, and superfluids have been effectively examined within the framework of the above approach.

The procedure for the topological classification has its roots in the idea that a condensed (ordered) medium is characterized by the order parameter $f(x)$ which is assumed to be a continuous map $f: K \to V$ from the physical space K occupied by the medium into the order parameter space V describing the internal symmetry properties of the medium. More precisely, V is a quotient space G/H, where G and H are taken to be groups of broken and unbroken symmetry of the medium, respectively. When studying the homotopic classification of maps $K \to V$ we come to the classification of the order parameter configurations – nonuniform states of the medium. Continuous deformations of the order parameter require a small amount of energy as contrasted to discontinuous ones. Therefore the topological classification of the order parameter configurations seems to be relevant and natural from the energetic point of view. The above statement is well supported by an experimental investigation of defects in ordered media [1–4].