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Ornstein–Zernike Behavior for Self-Avoiding Walks at All Noncritical Temperatures

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Abstract. We prove that the self-avoiding walk has Ornstein–Zernike decay and some related properties for all noncritical temperatures at which the model is defined.

1. Introduction

The original derivation of the long-distance behavior of a two-point correlation dates back to the classic work of Ornstein and Zernike [1] in 1914. Although Ornstein and Zernike examined only the classical fluid, it has since been realized that their conclusions should apply also to the two-point functions of many lattice spin systems and Euclidean quantum field theories. Moreover, by means of the Källen–Lehman representation, it has been demonstrated that there is a relatively straightforward relationship between the decay of the two-point function and the particle spectrum of the associated field theory (see, e.g. [13]). Motivated by this connection, there has been much interest in rigorously establishing Ornstein–Zernike decay for a variety of spin systems and lattice field theories [2–23]. Unfortunately, the vast majority of this work has established this decay only in a perturbative regime (e.g., high or low temperature or strong coupling).

In this paper, we consider self-avoiding walks, and prove Ornstein–Zernike decay and some related properties for all noncritical temperatures. Our method relies on the approach initiated in [24] and [2] (see also [9–11, 22, 23]), which shows that the original ideas of Ornstein and Zernike may be implemented whenever one can define a *direct correlation function* with a strictly larger decay rate than that of the two-point function. Here we prove such an assertion by constructing both a direct correlation function and a set of rescaled variables which bound this function. We then show that the rescaled variables obey a (renormalized) *Ornstein–Zernike inequality* which provides a bound on their decay rate and hence on that of the direct

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