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On Determinants of Laplacians on Riemann Surfaces

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Abstract. Determinants of Laplacians on tensors and spinors of arbitrary weights on compact hyperbolic Riemann surfaces are computed in terms of values of Selberg zeta functions at half integer points.

Introduction

In this paper we evaluate explicitly the determinants of Laplacians acting on arbitrary tensor and spinor fields on compact Riemann surfaces of constant negative curvature. They are equal to values of Selberg zeta functions at half-integer points, multiplied by an additional factor depending only on the genus and the weight of the field. Interest in such results comes from multiloop calculations for fermionic string theories and random surfaces, where these determinants arise from quantum fluctuations and Faddeev-Popov gauge fixing, while the extra factors can be viewed as finite corrections to the coupling constants [1].

Our approach is based on the explicit formulas for heat kernels of Fay [2], the Maass operators, and Selberg trace formulas. We observe that Selberg trace formulas have been used in similar contexts by many authors, notably Ray and Singer [3(a)], Donnelly [3(b)], McKean [4], Hejhal [5], Mandelstam [6], and Fried [7].

1. Tensors, Spinors, and Automorphic Forms

Let M be a compact Riemann surface with a fixed hermitian metric ds^2 of constant curvature -1, $\chi = 2-2h$ its Euler characteristic ($\chi < 0$), and let T^n denote the usual space of tensors $\{f(z)dz^n\}$ for n integer. If we fix a spinor structure among the 2^{2h} possible ones, we may also consider $n =$ (odd integer)/2, and view $T^{1/2}$ as the space of spinors, and T^n as spaces of spinor-tensor fields. Henceforth n will be allowed to take both integer and half-integer values. The covariant derivative ∇ sends T^n into

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