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# Bernoulli Property for a One-Dimensional System with Localized Interaction

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**Abstract.** We consider a one-dimensional system of particles on the half line $\mathbb{R}_+ = [0, \infty)$ interacting through elastic collisions among themselves and with a "wall" at the origin. On the first particle a constant force $E$ is acting, no external forces act on the other particles. All particles are identical except the first one which has a larger mass. We prove that if $E$ is such that the Gibbs equilibrium state exists, the corresponding equilibrium dynamical system is a Bernoulli flow.

## 1. Introduction

Consider the semi-infinite mechanical system consisting of a gas of infinitely many particles on the half line $\mathbb{R}_+ = [0, +\infty)$, interacting through elastic collisions with each other and with a "wall" at the origin. The mass of the first particle (i.e. the one closest to the origin) $M$ is assumed to be larger than the common mass $m$ of the other particles. A constant force $E > 0$ is acting only on the first particle (or "heavy particle," henceforth h.p.). The Gibbs equilibrium measure for all values of the temperature and the particle density such that $E < P$, where $P$ is the thermodynamic pressure that the gas exerts on the wall, is stationary in time. In [2], using techniques introduced in [1], it was proved that the corresponding dynamical system is a Bernoulli flow for $E < P/2$.

In this paper we extend the result to all values of $E < P$, by giving a simpler and more general proof. The main point is that instead of proving loss of memory by explicit probabilistic estimates, as in [1] and in [2], we make use of the following general features of the system: i) the fact that the invariant measure is locally absolutely continuous and the interacting subsystem (i.e. the h.p.) is confined, and ii) the local smoothness of the dynamics, i.e. the phase point of a finite particle system at time $t$ is "almost always" a smooth function of the initial data. Our methods of proof in their present form could be applied to a large class of systems

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