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Cluster Expansion for Abstract Polymer Models

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Abstract. A new direct proof of convergence of cluster expansions for polymer (contour) models is given in an abstract setting. It does not rely on Kirkwood-Salsburg type equations or "combinatorics of trees." A distinctive feature is that, at all steps, the considered clusters contain every polymer at most once.

1. Introduction

In the study of classical lattice models it is often useful to rewrite their partition function or their correlations in terms of polymer models. It is so whenever one is able to rewrite the partition function in a volume V in the form

$$Z(V) = \sum \prod_{\gamma \in \partial} \Phi(\gamma),$$

where the sum is over all compatible families ∂ of certain geometrical objects (γ) called polymers. A particular form of the polymer weights $\Phi(\gamma)$ depends on the considered model. See e.g. [1] for a collection of some typical examples. Notice that when applied to low temperature expansions the term contour is used instead of polymer.

If the weights $\Phi(\gamma)$ are small enough for "large" polymers γ, one may expand $\log Z(V)$ in a convenient form called cluster expansion, enabling one to make good evaluations of the "free energy" ("pressure") $\lim \frac{1}{|V|} \log Z(V)$ as well as of the decay of correlations. There are essentially two approaches to the proof of convergence of cluster expansion: one is based on the use of Kirkwood-Salsburg type of equations [2–5], while the other relies on "combinatorics of trees of a graph" [1, 6, 7]. In both these approaches many copies of one polymer may appear in a cluster. As a consequence the cluster expansion for $\log Z(V)$ is an infinite series even for finite V. Our goal here is to present a new formulation and a new proof of cluster expansions which is in our view both simple and straightforward, since it uses, at all stages, only clusters in which every polymer may appear at most once.