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Random Fields on Riemannian Manifolds: A Constructive Approach

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Abstract. We extend to "Euclidean" fields on a wide class of Riemannian manifolds two results which have proven to be crucial in the construction of interacting quantum fields in the flat case, namely local regularity properties of the free covariance in two dimensions and Osterwalder-Schrader positivity.

1. Introduction and Outline

The peculiarity and the depth of the physical problems posed by Quantum Field Theory on a gravitational background [1] and the need of providing models of existing axiomatic proposals [2, 3], motivate the opportunity of a constructive approach.

In this paper we extend some techniques of constructive quantum field theory to the case of two dimensional properly Riemannian ("Euclidean") manifolds.

The specific case of the two dimensional sphere (vs. two dimensional de Sitter universe) was considered in [4]. Here we rather rely on general intrinsic properties of the heat kernel on the manifold as the main tool to overcome the difficulties due to the non-availability of explicit expressions for the free Green's functions.

Our arguments which, because of the lack of translational invariance, must do without the tool of Fourier transformation, shed, we hope, some light also on the classical arguments for the conventional flat case.

The "Euclidean" approach of [1] is exceedingly convenient in the case (which we will always be considering in the following) of a paracompact, complete, C^∞ Riemannian manifold M.

As pointed out in [5], the uniqueness of the free covariance $C=(-\Delta_M+m^2)^{-1}$ (Δ_M being the Laplace-Beltrami operator on M), which follows from the essential self-adjointness of Δ_M in $C_0^\infty(M)$ [6] gives an unambiguous starting point for the "Euclidean" construction, as opposite to the ambiguities arising in the choice of a free vacuum on a manifold of Lorentzian signature [7].

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