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Surface Effects in Debye Screening*

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Abstract. A thermodynamic system of equally charged, plus and minus, classical particles constrained to move in a (spherical) ball is studied in a region of parameters in which Debye screening takes place. The activities of the two charge species are not taken as necessarily equal. We must deal with two physically interesting surface effects, the formation of a surface charge layer, and long range forces reaching around the outside of the spherical volume. This is an example in as much as 1) general charge species are not considered, 2) the volume is taken as a ball, 3) a simple choice for the short range forces (necessary for stability) is taken. We feel the present system is general enough to exhibit all the interesting physical phenomena, and that the methods used are capable of extension to much more general systems. The techniques herein involve use of the sine-Gordon transformation to get a continuum field problem which in turn is studied via a multi-phase cluster expansion. This route follows other recent rigorous treatments of Debye screening.

0. Introduction

The rigorous study of Debye screening was initiated in [4] by Brydges, with the treatment of a charge symmetric lattice Coulomb gas. This work was greatly generalized by Brydges and Federbush [7]. Their proof applies to continuum Coulomb systems with essentially arbitrary short range forces, and charge symmetry is not required. Imbrie [12] improved the convergence estimates of [7] and removed a restriction on the relative sizes of the activities. He also proved Debye screening in Jellium.

All of these treatments of Debye screening impose two important constraints on the system. First, there is a constraint on the activities z_i and charges e_i which is usually referred to as a "neutrality" condition. This condition may be viewed as essentially saying that $\sum_i z_i e_i = 0$. Second, Dirichlet boundary conditions for the

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