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Superconformal Current Algebras and Their Unitary Representations

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Abstract. A natural supersymmetric extension $(\widehat{dG})_\kappa$ is defined of the current (= affine Kac–Moody Lie) algebra $\widehat{dG}$; it corresponds to a superconformal and chiral invariant 2-dimensional quantum field theory (QFT), and hence appears as an ingredient in superstring models. All unitary irreducible positive energy representations of $(\widehat{dG})_\kappa$ are constructed. They extend to unitary representations of the semidirect sum $S_\kappa(G)$ of $(\widehat{dG})_\kappa$ with the superconformal algebra of Neveu–Schwarz, for $\kappa = \frac{1}{2}$, or of Ramond, for $\kappa = 0$.

0. Introduction

The semidirect sum of the Virasoro algebra W_c and the algebra $\widehat{dG}$ of left (or right) currents for a compact Lie group G arises naturally in both conformal invariant 2-dimensional QET models [1–3] and in the general study of infinite dimensional Lie algebras [4–7] (see also [8, 9]). Its supersymmetric extension which is implicit in recent work on superstrings [10–12] also admits a local field interpretation (partly exploited in [13, 14] as a development of the QFT approach of [15]).

The objective of this note is two-fold: (a) to set a mathematical framework in which the supercurrent and string superalgebras arise naturally; (b) to classify all hermitian (= unitary) positive energy representations of these algebras. A remark is also included, concerning the unitarity of the discrete series of representations of the super Virasoro algebra (with central charge $c < \frac{3}{2}$).

In the theory of infinite dimensional Lie algebras a chiral current algebra $\widehat{dG}$ (called an *affine Kac–Moody algebra*) arises as a central extension of the *loop algebra* $\widetilde{dG}$ generated by tensor products of elements of the finite dimensional Lie algebra dG with Laurent polynomials of a complex variable t. The supersymmetric extensions

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