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Ultraviolet Stability of Three-Dimensional Lattice Pure Gauge Field Theories*

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Abstract. We prove the ultraviolet stability for three-dimensional lattice gauge field theories. We consider only the Wilson lattice approximation for pure Yang–Mills field theories. The proof is based on results of the previous papers on renormalization group method for lattice gauge theories.

Introduction

In this paper we give the first, simplest application of the results of [2]–[7]. In these we have developed the renormalization group approach to gauge field theories, in the form proposed by K. Wilson in [24, 25]. We prove here ultraviolet stability of lattice approximations to three-dimensional pure Yang–Mills field theories. We give a simple proof, following the method of [8]–[10] on three-dimensional Higgs model, i.e. a proof using some special features of superrenormalizable models, but we use also Wilson's ideas on the role of scaling transformations. More exactly in lattice gauge theories we use improvement of regularity properties of typical gauge fields instead of non-existing scalings. In fact we use these ideas in the proof of finiteness of the resulting effective theory, and this aspect of the proof is non-perturbative, although we use the superrenormalizability and perturbative expansions to produce this effective theory. This makes the proof especially simple and short. It is based on almost all results of the papers [2]–[7], and we assume that the reader is familiar with these, especially with notations and definitions. Reading [1] is recommended for those who want to get enough information to read the present paper independently of the others.

An awkward and difficult aspect of the paper is a formulation of results. At the beginning we start with a general and simple formulation, and then we make it more precise when we develop our method. Thus let us explain at first in very general terms what we understand by the ultraviolet stability. This notion is strictly connected

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