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How Different are the Supermanifolds of Rogers and DeWitt?

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Abstract. A DeWitt supermanifold always has the structure of a vector bundle over an ordinary spacetime manifold, whereas a Rogers supermanifold is not so restricted. Corresponding to the vector space fibers of the DeWitt supermanifold, a Rogers supermanifold has a foliation by submanifolds, or leaves, parametrized by soul coordinates only. We show that the universal covering space of any leaf always admits a flat metric. If the covering space is complete in this metric, it must in fact be a vector space. We combine this result with known theorems about foliations to give conditions under which a compact Rogers supermanifold with a single even dimension is necessarily a quotient space of flat superspace. We also show that a supermanifold defined by a polynomial equation in flat superspace is always of the DeWitt type. Finally, we exhibit new supermanifold structures for R^2 and the 2-torus which show that the foliation of a Rogers supermanifold can be quite exotic.

1. Introduction

Just as general relativity is best formulated in terms of the differential geometry of a four-dimensional spacetime manifold, so the mathematical structure of supergravity is best understood as the differential geometry of a "supermanifold" having both commuting and anticommuting coordinates. Although the supergravity literature focuses on local geometry, rigorous mathematical definitions of supermanifolds have been given which permit the study of global topological questions as well. Of these, the most general definition is that of Rogers [1]. A Rogers supermanifold possesses only the minimal structure needed to define superfields with the properties required in supersymmetric theories. DeWitt's definition [2] is more restrictive, but still adequate for physical applications.

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