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Renormalization Theory in Four Dimensional Scalar Fields (II)

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Abstract. We interpret the results of the preceding paper (I) in terms of partial resummations of the perturbative series for the effective interaction. As an application we sketch how our resummation method leads to a simple summation rule leading to a convergent expansion for the Schwinger functions of the planar Φ^4_4-theory.

1. Introduction

In the preceding paper [Commun. Math. Phys. **100**, 545–590 (1985), referred to here as (I)] we have shown how to find "$n!$-bounds" on the perturbative coefficients of the effective potential and of the Schwinger functions. The method is based on a renormalization group approach to renormalization theory. In this paper we show that there is a natural way of collecting together various perturbative contributions through a general analysis of summation rules for divergent series. For each resummation we introduce some recursive relation between formal power series so that the resummation is by definition a non-formal solution of the same recursive relation. The similarity of this procedure with the non-perturbative methods based on the "beta function" is manifest. As applications we discuss the theory of the leading contribution to the effective potentials at momentum p as $p \to \infty$ and the convergence of the planar Φ^4_4-theory sketching a simple proof of the theorems of 't Hooft and Rivasseau [13].

Another aim of this paper is to provide some technical details not explicitly presented in (I); they have all been collected in Appendix A. In this paper the formulae preceded by a I [e.g. (I, 2.10)] refer to the paper (I).

Many of the ideas appearing in this paper overlap with those of references [1–14].

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