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Large Deviations from Classical Paths. Hamiltonian Flows as Classical Limits of Quantum Flows

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Abstract. We prove that in the limit $\hbar \to 0$, the probability for the paths of the stochastic jump process associated to the quantum time evolution to be in a tublet around the classical trajectory is of order $1 - \exp\{-A/\hbar\}$. We give some applications of this result to the study of the classical limit of Wigner functions.

1. Introduction

In a previous paper [1] it was shown that the real time evolution of typical matrix elements of a relativistic quantum field theory with trigonometric interaction can be described in any space time dimension by a stochastic flow on the function space of initial conditions. More precisely, there exists a generalized stochastic process $(\Phi(x,t), \Pi(x,t))$ with value in the space of initial conditions and a functional S of this process such that the expectation value at time t in the ground state of the exponential of the field operator is given by

$$\begin{aligned}&(\Omega\,|\exp\{i(\Phi_t f) - i(\Pi_t q)\}\Omega)\\ &\quad = e^{ct}\,\mathbb{E}\left[\exp\left\{\frac{i}{h}S_t\right\}(\Omega\,|\exp(i(\Phi\,\Phi_t) - i(\Pi\,\Pi_t))\Omega)\restriction_{t=0}\right],\end{aligned} \tag{1.1}$$

c being a constant. This expression turned out to be very convenient to prove the existence of limits when the cutoffs required to define the interaction are removed.

In a second paper [2] we concentrated on the case of quantum mechanics, viz on the case of systems with a finite number of degrees of freedom. We have shown that it was more natural and useful to write the previous expression using a process in a

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