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Large Time Behaviour of Some N-Body Systems*

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Abstract. In this work we prove completeness for N-body systems that evolve asymptotically into either N free particles or a two cluster system with one of the clusters being a single particle. For the three body case our results imply completeness for a very general system with potentials decaying like $|x|^{-1-\varepsilon}$ at ∞.

Introduction

Completeness in many body scattering was first prove by Faddeev for three particles using a time-independent method in [1] and was followed up by many authors for the same case. All these methods are limited in that they make assumptions on the spectral properties of the subsystems. For an excellent review of these results see Ginibre [2]. On the other hand for the N-body short range systems Lavine [3] proved a completeness result when the potentials are repulsive, Iorio-O'Carroll [4] proved it when the potentials are weakly coupled and Sigal ([5] and the references given there) proved completeness for a class of generic short range potentials.

More recently Enss has continued his work on time-dependent scattering theory that he pioneered for the two body case to three body systems. In [6] he treated two cluster scattering of N-body systems. While in [7] he gave a rough sketch of the proof of three-body completeness, in [8] he made the proof clearer. See [8a] for a complete proof of the three-body long and short range cases. In [9] he has a slightly different approach to the proof and also some discussion of the general case. Almost simultaneously with [7, 8], Sinha et. al. [10] proved three-body completeness for pair potentials with $(2+\varepsilon)$ decay at ∞. They also incorporated some ideas of Enss in their work. See the work of Kitada for a different approach [11].

Mourre on the other hand has used his work on the spectral theory of many body operators [12] to determine some propagation properties of the N-body total evolution in some weighted spaces. He obtained L^2 estimates [13] in certain

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