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Bounds on Renormalized Feynman Graphs

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Abstract. We prove two bounds on the value of renormalized Euclidean Feynman graphs. One is a relatively crude but widely applicable bound; the other a finer bound applicable to ϕ_4^4-like models.

I. Introduction

In [FMRS 1] we used a phase space expansion to prove an exponential bound on completely convergent Euclidean Feynman graphs:

$$|G| \leqq K^{L(G)}, \tag{1.1}$$

where $|G|$ is the value of the amplitude associated to the graph G and $L(G)$ is the number of lines in the graph. In this paper K will stand for various numerical constants whose precise values are largely irrelevant.

In this companion paper we apply the same expansion to the technically more difficult cases of renormalized graphs.

The bound (1.1) fails for renormalized graphs in strictly renormalizable theories; explicit families of graphs with factorial growth have been constructed [GN1, L]. On the other hand in [CR], a general bound on graphs of massive ϕ_4^4 has been derived, which grows as the factorial of the number of renormalization subtractions. It extends to renormalized graphs of massless ϕ_4^4 at non-exceptional momenta [CPR]. These bounds are not optimal.

For example, if

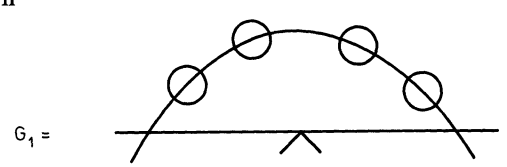

Fig. 1.1

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