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Spontaneous Symmetry Breaking and Energy Gap Generated by Variables at Infinity*

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Abstract. Spontaneous symmetry breaking in the presence of long range instantaneous interactions is studied and the general mechanism underlying it is clarified. A characteristic feature is that the algebraic dynamics does not leave any essentially local algebra stable, i.e. variables at infinity get involved in the time evolution of local variables, so that in each irreducible representation the time evolution fails to be symmetric. For continuous symmetries, the Fourier transform of the vacuum expectation value of charge commutators is related to the energy spectrum at low momenta and a generalized Goldstone theorem is proved which explains the generation of energy gap. This energy gap is further shown to be governed by a "classical dynamics at infinity", equivalently by the group generated by the effective Hamiltonian and the charge. Explicit examples are discussed.

1. General Questions about Spontaneous Symmetry Breaking and Energy Gap

The phenomenon of spontaneous symmetry breaking appears to be at the basis of several collective effects in many body physics and it plays a crucial role in the unification of elementary particle interactions. Soon after the realization of such a mechanism, considerable interest was devoted to the characterization of the general structures underlying it. The aim was to get both insight on constraints involved in the use of such a mechanism and non-trivial dynamical information on essentially non-linear (collective) effects, without relying on approximations and/or perturbative expansions. The strongest of such characterizations was provided by Goldstone's theorem [1], but it was soon realized that a precise characterization of the hypotheses was crucial for the applicability of the theorem. Actually, from the many examples of many body systems and from elementary particle physics, it became clear that it was important to isolate possible symmetry breaking mechanisms, which allow the evasion of the Goldstone theorem. The

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