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Some Comments on the Non-Self-Dual Nahm Equations

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Abstract. Some empirical results on the cubic algebra $2a_i = \sum_j [a_j, [a_j, a_i]]$ are presented. The algebra is satisfied at the residue of any pole in a solution to Nahm's non-self-dual equations.

1. Introduction

Recently, there has been some interest in Yang-Mills equations in one dimension, reduced from four by insisting that all fields be independent of three of the variables. On the one hand, some work has been done on the mechanical system which results when the four dimensional theory is defined in Minkowski space and the reduction is simply to delete all dependence on the cartesian spatial variables $\mathbf{x}$ [1]. This mechanical system has been shown to be essentially chaotic by Nikolaevskii and Schur and by Savviddy [2]. On the other hand, if we take the Euclidean version of the four dimensional theory and reduce it by deleting the dependence on any three of the cartesian coordinates, we obtain a set of equations, differing from the previous set only by a single sign but which is interesting in a very different context. In this case, certain solutions to the self-dual equations, which imply the second order equations, of course, have a remarkable role to play. They are intimately related via the ADHMN [3] (Atiyah-Drinfeld-Hitchin-Manin-Nahm) construction to static monopole solutions to Yang-Mills-Higgs theories in four dimensions in the Bogomol'nyi-Prasad-Sommerfield limit [4]. Indeed, the ADHMN construction may be viewed [5] as an equivalence between two sets of self-dual equations (with appropriate boundary conditions), one as described above in one dimension, the other in three dimensions (reduced from a Euclidean four dimensional theory by deleting dependence on a single variable); the latter corresponding to the equations satisfied by the vector potential and Higgs' field in the BPS limit.

It is known that the SU(2) Yang-Mills-Higgs theory has solutions of finite energy which do not satisfy the first order self-dual equations but are, nevertheless, solutions to the second order field equations [6]. Taubes' existence proof of this

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