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Topological Charges in Monopole Theories

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Abstract. Let us consider a monopole theory with a compact, simply connected gauge group and the Higgs field in the adjoint representation. Using root theory we show that.

(i) The homotopy class of the Higgs field is a p-tuple of integers where p is the dimension of the centre of the residual symmetry group. These "Higgs charges" can be expressed as surface integrals of differential forms.

(ii) To any invariant polynomial on the Lie algebra is associated a topological invariant which turns out to be a combination of the Higgs charges.

(iii) Electric charge is quantized. The monopole's magnetic charge is a combination – with the Higgs charges as coefficients – of p basic magnetic charges which satisfy generalized Dirac conditions.

The example of $G = \mathrm{SU}(N)$ is worked out in detail.

1. Introduction

The fundamental role played by topological invariants in monopole theory has been recognized since the very beginning [1, 2, 3, 18]. Essentially three types of such "charges" have been considered so far:

(i) Assume that the full gauge group G is broken spontaneously to H by the vacuum expectation value of the Higgs field Φ. The usual requirements concerning the asymptotic behaviour of Φ imply the existence of a map from S^2, the 2-sphere at infinity, into an orbit G/H of G. We have thus a homotopy class

$$[\Phi] \in \pi_2(G/H). \tag{1.1}$$

This first topological invariant shall be referred to as the Higgs charge.

(ii) Let F denote the field strength tensor. If the Higgs field is covariantly constant, the quantity

$$I = \int_{S^2} \mathrm{tr}(F \cdot \Phi) \tag{1.2}$$