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# On the Theory of Recursion Operator

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**Abstract.** The general structure and properties of recursion operators for Hamiltonian systems with a finite number and with a continuum of degrees of freedom are considered. Weak and strong recursion operators are introduced. The conditions which determine weak and strong recursion operators are found.

In the theory of nonlinear waves a method for the calculation of the recursion operator, which is based on the use of expansion into a power series over the fields and the momentum representation, is proposed. Within the framework of this method a recursion operator is easily calculated via the Hamiltonian of a given equation. It is shown that only the one-dimensional nonlinear evolution equations can possess a regular recursion operator. In particular, the Kadomtsev–Petviashvili equation has no regular recursion operator.

## I. Introduction

The inverse scattering transform method gives a possibility of investigating in detail a wide class of both the ordinary and partial differential equations (see e.g. [1–3]). The equations, integrable by the inverse scattering transform method, possess a number of remarkable properties: solitons, infinite sets of conservation laws, infinite symmetry groups, complete integrability, etc. In turns out also that the equations, to which the inverse scattering transform method is applicable, have the pronounced recursion structure. The so-called recursion operator plays a central role in the formulation of these recursion properties. The role of the recursion operator is two-fold. Firstly, it allows one to write out the families of equations integrable by a given spectral problem in a compact form. For example, the family of equations connected with the famous Korteweg–de Vries (KdV) equation can be represented as follows:

$$\frac{\partial u(x,t)}{\partial t} + \partial L^n u = 0\,, \tag{1.1}$$