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Random Media and Eigenvalues of the Laplacian

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Abstract. Let β be a fixed number >1. We remove $[m^\beta]$-balls of centers $w_1, \ldots, w_{[m^\beta]}$ with the same radius α/m from a bounded domain Ω in $\mathbf{R}^3$. We consider the asymptotic behaviour of the k^{th} eigenvalue of the Laplacian in $\Omega\backslash\overline{[m^\beta\text{-balls}]}$ under the Dirichlet condition as a random variable on a probability space $(w_1, \ldots, w_{[m^\beta]}) \in \Omega^{[m^\beta]}$, when $m \to \infty$.

1. Introduction

In the present note we consider a mathematical problem concerning random media. We consider a bounded domain Ω in $\mathbf{R}^3$ with smooth boundary Γ. We put

$$B(\varepsilon; w) = \{x \in \mathbf{R}^3; |x-w| < \varepsilon\}.$$

Fix $\beta \geqq 1$. Let $0 < \mu_1(\varepsilon; w(m)) \leqq \mu_2(\varepsilon; w(m)) \leqq \ldots$ be the eigenvalues of $-\Delta\,(=-\text{div-grad})$ in $\Omega_{\varepsilon, w(m)} = \Omega \Big/ \overline{\bigcup_{i=1}^{\tilde{m}} B(\varepsilon; w_i^{(m)})}$ under the Dirichlet condition on its boundary. Here $\tilde{m}$ denotes the largest integer which does not exceed m^β, and $w(m)$ denotes the set of $\tilde{m}$-points $\{w_i^{(m)}\}_{i=1}^{\tilde{m}} \in \Omega^{\tilde{m}}$. Let $V(x) > 0$ be C^1-class function on $\bar{\Omega}$ satisfying

$$\int_\Omega V(x)dx = 1\,.$$

We consider Ω as the probability space with the probability density $V(x)dx$. Let $\Omega^{\tilde{m}} = \prod_{i=1}^{\tilde{m}} \Omega$ be the probability space with the product measure. The following result which is an elaboration of Kac's theorem (Kac [3]) was given in Ozawa [5].

Theorem A. *Assume that* $\beta = 1$. *Fix* $\alpha > 0$ *and* k. *Then,*

$$\lim_{m \to \infty} \mathbb{P}(w(m) \in \Omega^{\tilde{m}}; \quad m^{\tilde{\delta}}|\mu_k(\alpha/m; w(m)) - \mu_k^V| < \varepsilon) = 1$$

holds for any $\varepsilon > 0$ *and* $\tilde{\delta} \in [0, 1/4)$. *Here* μ_k^V *denotes the* k^{th} *eigenvalues of* $-\Delta + 4\pi\alpha V(x)$ *in* Ω *under the Dirichlet condition on* Γ.