Commun. Math. Phys. 92, 309–327 (1984)

Communications in
Mathematical
Physics

The Spectrum of the Transfer Matrix in the C^*-Algebra of the Ising Model at High Temperatures

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Abstract. We investigate the state on the Fermion algebra which gives rise to the thermodynamic limit of the Gibbs ensemble in the two-dimensional Ising model on a half lattice with nearest neighbour interaction. It is shown that the operator P_∞^- in the GNS space, which performs the essential functions of the renormalized transfer matrix, has a quasi-particle structure.

1. Introduction

In lattice models with an interaction potential of finite range, the free energy in a finite volume is determined by the largest eigenvalue of a matrix, known as the transfer matrix. One question which naturally arises is how to normalize the transfer matrix so that it becomes a well-defined operator in the thermodynamic limit. Such a renormalization is easy to make in the domain of Gibbs-state uniqueness (Minlos and Sinai [19]). The limit in this case is a stochastic operator which has a property of asymptotic multiplicativeness which suggests the conjecture that the spectrum of the operator has a quasi-particle structure: there is a grading of the Hilbert space on which the stochastic operator acts into subspaces corresponding to different sets of quasi-particle occupation numbers; these subspaces are invariant under the action of the stochastic operator; on these subspaces the stochastic operator has a simple structure and acts by multiplication. A general analysis of the spectral properties of a stochastic operator arising from a transfer matrix was undertaken by Minlos and Sinai [19] who contructed the single-particle subspace assuming a cluster-property of the transfer-matrix. The first proof of this cluster-property for the two-dimensional Ising model with nearest neighbour interactions was provided by Abdulla-Zade et al. [1]. Malyshev [14, 15] used cluster expansions to make improved estimates of matrix elements and which enabled him to work in arbitrary dimensions, Malyshev and Minlos [17, 18] used these estimates to prove that, for sufficiently small values of β, an operator with the cluster-property has invariant subspaces which are reminiscent of the n-particle subspaces of Fock space; the restriction of the operator to the