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The Order Parameter in a Spin Glass

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Abstract. Various possible precise definitions of an Edwards-Anderson type of order parameter for an Ising model spin glass are considered, using boundary conditions for a finite system, states of an infinite system, and a duplicate-system approach. Several of these definitions are shown to yield identical results.

I. Introduction

It is generally believed that if spin glass ordering can be described within the framework of equilibrium statistical mechanics, it corresponds to a non-zero value of the Edwards–Anderson order parameter [1]

$$q_{EA} = \langle \langle S_i \rangle^2 \rangle, \tag{1}$$

where S_i is the magnetization at site i, the inner brackets refer to a thermal average, and the outer brackets an average over an ensemble of random systems. As it stands, this definition is ambiguous, since it will depend, in general, on the choice of boundary conditions for finite random systems.

For a random Ising model (Sect. II) we consider some alternative definitions (Sect. III) which have the virtue of being well-defined in the thermodynamic limit. One is a "thermodynamic" definition [2], one is based on states of an infinite system, and one on the maximization of a quantity like (1) over all possible boundary conditions. We show (Sect. IV) that in the thermodynamic limit they are all well-defined and equal to each other. For simplicity of exposition, the discussion is limited to zero magnetic field, but its extension to non-zero field (Sect. V) causes no difficulty. Yet another rather appealing definition (Sect. VI) encounters some technical difficulties we have not resolved.

II. Spin-Glass Model

For simplicity of exposition and in order to avoid having to state a host of technical restrictions, we limit our discussion to a random Ising model $S_i = \pm 1$ with

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