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Spin Waves, Vortices, and the Structure of Equilibrium States in the Classical XY Model

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Abstract. We prove that, for spin systems with a continuous symmetry group on lattices of arbitrary dimension, the surface tension vanishes at all temperatures. For the classical XY model in zero magnetic field, this result is shown to imply absence of interfaces in the thermodynamic limit, at arbitrary temperature. We show that, at values of the temperature at which the free energy of that model is continuously differentiable, i.e. at all except possibly countably many temperatures, there is *either* a *unique translation-invariant equilibrium state, or all* such states are labelled by the elements of the symmetry group, SO(2). Moreover, there are *no non-translation-invariant, but periodic* equilibrium states. We also reconsider the representation of the XY model as a gas of spin waves and vortices and discuss the possibility that, in four or more dimensions, translation invariance may be broken by imposing boundary conditions which force an (open) vortex sheet through the system. Among our main tools are new correlation inequalities.

1. Introduction

In this paper we investigate the structure of the equilibrium states of the classical XY model in zero magnetic field.[1] It is well-known that there is only one translation invariant equilibrium state in two dimensions [1, 2]. There is however a phase transition (the Berezinskii-Kosterlitz-Thouless transition). A mathematical proof thereof has been given in [3]. In three or more dimensions the XY model exhibits spontaneous magnetization and symmetry breaking at sufficiently low temperatures [4, 5]. In this paper we give a complete description of all extremal, translation invariant equilibrium states, i.e. the pure phases, whenever the free energy is differentiable with respect to the temperature (Sect. 3.3): We prove that either the set of pure phases is in one-to-one correspondence with the internal symmetry group, or else there is only one translation invariant state. We use new

1 This model is of interest in connection with the theory of superfluid Helium and with superconductivity, for example