

A Phase Cell Cluster Expansion for a Hierarchical ϕ_3^4 Model*

Guy A. Battle III** and Paul Federbush

Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA

Abstract. The formalism developed in a previous paper is applied to yield a phase cell cluster expansion for a hierarchical ϕ_3^4 model. The field is expanded into modes with specific renormalization group scaling properties. The present cluster expansion for a vacuum expectation value is formally the natural factorization of each term in the perturbation expansion into the contribution of modes connected to the variables in the expectation via interactions, and that of the complementary set. The expectation value is thus realized as a sum of contributions due to *finite* subsets of the modes. We emphasize the following additional features:

- 1) Partitions of unity are not used.
- 2) There are *essentially* no cut-offs.
- 3) The expansion is developed directly, without an initial need to prove an ultraviolet stability bound, the most difficult part of the traditional approach.

Our main interest in the present phase cell cluster expansion is founded in the belief that it may be the right vehicle for proving the existence of a non-trivial four-dimensional field theory.

0. Introduction

Techniques developed in the study of ϕ_3^4 should eventually be useful in other directions – in statistical mechanics, in fluid mechanics, in the study of turbulence. We here restrict our sights to further applications in field theory, in particular to the construction of non-trivial four-dimensional field theories. For us the study of ϕ_3^4 is taken in this light. For convenience we divide the bulk of work on ϕ_3^4 into five tracks.

1) The first important contribution was the establishment of the ultraviolet bound by Glimm and Jaffe [9]. This most difficult paper indicated the importance

* This work was supported in part by the National Science Foundation under Grant No. PHY 79-05688

** On leave from Mathematics Department, Texas A&M University, College Station, TX 77843, USA