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Debye Screening for Jellium and Other Coulomb Systems

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Abstract. Debye screening is proven for a large class of classical Coulomb gases at low densities. Among the models treated are jellium systems (where particles interact with a fixed background charge), systems with arbitrarily dilute fractional charges, and systems where the charges are not integrally related. The interaction potentials of the corresponding sine-Gordon models may have no symmetry and can have infinitely many stationary points which are degenerate or nearly degenerate in energy.

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Introduction

The classical Coulomb gas has been the subject of several rigorous investigations in the last few years. Brydges [1] established Debye screening for a lattice Coulomb gas. His work was greatly generalized by Brydges and Federbush [3] who considered the continuous statistical mechanics situation with a large class of

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