Commun. Math. Phys. 87, 417–427 (1982)

Communications in Mathematical Physics

Infrared Bounds and the Peierls Argument in Two Dimensions*

Jean Bricmont and Jean-Raymond Fontaine

Institut de Physique Théorique, Université Catholique de Louvain, Chemin du Cyclotron, B-1348 Louvain-la-Neuve, Belgium

Abstract. We propose a definition of contours for spin systems which leads to improved estimates on the region of parameters where several phases coexist. We discuss as examples anisotropic rotators and a $\lambda\phi^4$ lattice field theory. Our contours are estimated using infrared bounds and they are related to those of Euclidean Field Theory.

I. Introduction

The purpose of this paper is to present improved estimates on the region of parameter space where phase coexistence takes place. We discuss few examples but the method based on a new (at least in statistical mechanics) definition of contours may have a wider range of applicability.

Specifically we first consider (Sect. 2) an anisotropic rotator model in two dimensions:

$$-H = \sum_{\langle xy \rangle} \mathbf{S}_x \cdot \mathbf{S}_y + \alpha \sum_{\langle xy \rangle} S_x^1 S_y^1 . \tag{1}$$

It is known [9, 13, 14] that for any $\alpha \neq 0$ there is a spontaneous magnetisation for β large enough. However, as α goes to zero, β has to be taken of order α^{-1} (at least). Heuristic arguments indicate that this is not the actual behaviour of $\alpha_{\text{crit}}(\beta)$: as β goes to infinity $\alpha_{\text{crit}}(\beta)$ should behave like $\exp(-c\beta)$ except for the case of two components where, due to the Kosterlitz-Thouless transition in the $\alpha = 0$ case [11], $\alpha_{\text{crit}}(\beta)$ should reach zero for finite β.

We prove that for some $c > 0$, if $\alpha = \exp(-c\beta)$ then for all β large enough, the model (1) exhibits a spontaneous magnetisation. However our constant c is not best possible.

---

* Part of this work was done while Jean Bricmont was at the Mathematics Department of Princeton University and Jean-Raymond Fontaine was at the Mathematics Department of Rutgers University. This work was supported by NSF Grant No. MCS 78-01885 at Princeton University and by NSF Grant No. PHY 78-15920 at Rutgers University

0010-3616/82/0087/0417/$02.20