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Existence and Uniqueness for Random One-Dimensional Lattice Systems

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Abstract. Existence and uniqueness are shown for the fixed point problem pertinent to hopping transport in one-dimension with random transfer rates. Continuity properties of the solution are exhibited. The connection with Dyson's treatment of the linear harmonic chain with random masses is established.

1. Introduction

Diffusion or hopping transport on the one-dimensional lattice $\mathbb{Z}$ is described by the master equation

$$\dot{P}_n = W_{n-1}(P_{n-1} - P_n) + W_n(P_{n+1} - P_n), \tag{1.1}$$

where $P_n(t)$ is the probability of finding a particle at time t on the lattice site n. Randomness is introduced by assuming the transfer rates W_n, $n \in \mathbb{Z}$, to be independent $\mathbb{R}_+$-valued random variables, equally distributed according to a probability measure ν. Thus, one is lead to consider expectations

$$E(f) = \int \prod_{n \in \mathbb{Z}} d\nu(w_n) f(\{w_n\}) \tag{1.2}$$

of measurable functions f on $\mathbb{R}_+^{\mathbb{Z}}$. In [1] it has been shown (by supplementing (1.1) with the initial condition $P_n(0) = \delta_{n0}$), that

$$E(\tilde{P}_0(s)) = \int_0^\infty dt e^{-st} E(P_0(t)) \tag{1.3}$$

is given by

$$E(\tilde{P}_0(s)) = \iint_{\mathbb{R}_+^2} d\mu_s(x) d\mu_s(y) (x + y + s)^{-1} \tag{1.4}$$

for $s \geqq 0$. Here, μ_s, $s \in \mathbb{R}_+$, is a probability measure on $\mathbb{R}_+$ satisfying the integral equation

$$\mu_s([0, x)) = \iint_{A_{s,x}} d\nu(y) d\mu_s(z), \quad x > 0, \tag{1.5}$$

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